Diffuser Tracer Test Interpretation

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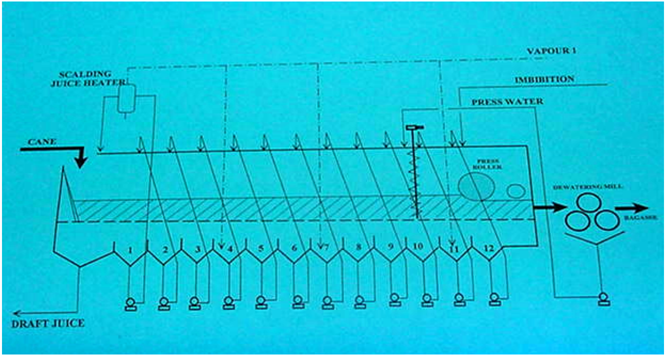
Description of the problem



Cane enters at left, exits right
Water/juice enters at right, exits left

Description of the problem

- Fibre flows through the diffuser (left to right)
- Juice flows from spray jets onto top, flowing through to be collected in trays beneath.
- Collected water is pumped upstream (in fibre) and re-sprayed
- ... but from where does the juice in each tray come?
- Experiments inputing a tracer into one of the bins were conducted ..
- **MISG:** Can we interpret tracer data from "dumped" salt water input?
- **MISG:** Produce a management-generic-interpreter in a Spreadsheet....

Underlying assumption - Model

- Treat the sugar fibres as porous medium ....
- Define *Piezometric Head* $\phi = \frac{p}{\rho g} + y$
- Apply Darcy's Law $\mathbf{q} = (u, v) = -\kappa \nabla \phi$
- Combine with conservation of volume $\nabla \cdot \mathbf{q} = 0 \Rightarrow \nabla \cdot \kappa \nabla \phi = 0$
- If κ (permeability) is constant

$$\Rightarrow \nabla^2 \phi = 0$$

- Boundary conditions - on surface $y = \eta(x, t)$

$$\phi = \frac{p(x,t)}{\rho g} + \eta, \quad \text{on} \quad y = \eta(x,t)$$

- Kinematic condition

$$\eta_t + u\eta_x - v = 0 \quad \text{on} \quad y = \eta(x,t)$$

- Dynamic condition at bottom

$$\phi = 0 \quad \text{on} \quad y = 0.$$

Simple linear solution

Assume the sprays are sinusoidal (just to get quick solution), so

$$\frac{p}{\rho g} = A - A\sin(\frac{\pi x}{L} - ct)$$

where c is speed of the bed, L is the distance between sprays (length of the trays) and A is the amplitude of the pressure of the sprays.

$$\text{Assume } \phi = \int_0^{\infty} \sinh(ky)(a(k,t)\sin(kx) + b(k,t)\cos(kx))dk$$

$$\text{and linearize about } y = D \Rightarrow \phi(x, D, t) = A - A\sin(\frac{\pi x}{L} - ct) + y$$

$$\text{and } \eta_t = -K\phi_y(x, D, t)$$

$\phi = 0$ on $y = 0$ is satisfied. The solution is

$$\phi(x, y, t) = A\left[\frac{\sinh\frac{\pi y}{L}}{\sinh\frac{\pi D}{L}}\right]\sin(ct - \frac{\pi x}{L})$$

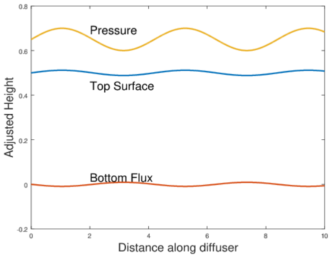
Simple linear solution (ctd)

The solution for $\eta(x,t)$ is

$$\eta(x,y,t) = D - A\frac{\pi K}{L}\left[\frac{\cosh\frac{\pi D}{L}}{\sinh\frac{\pi D}{L}}\right]\sin(ct - \frac{\pi x}{L})$$

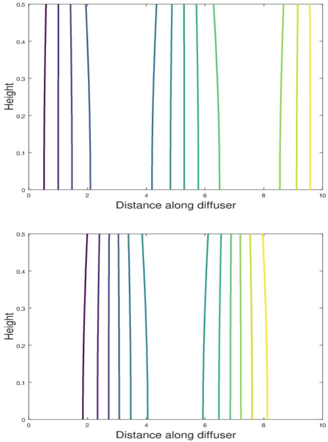
and can work out the streamlines as

$$\psi(x,y,t) = x + A\left[\frac{\cosh\frac{\pi y}{L}}{\sinh\frac{\pi D}{L}}\right]\cos(ct - \frac{\pi x}{L})$$



Streamlines t=0, t=6

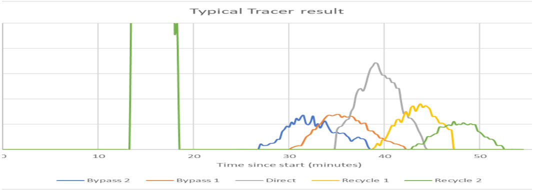
Streamlines at $t = 0$ and $t = 6$.



Close together means faster flow as the pressure pulse (spray) passes overhead.

Experiment

- 50 kg salt added to 200 litres of water and injected into the pipe before one of the inlet sprays ....
  Density of $1.03 kg/m^3$ *significantly higher than juice(?) so is it just a tracer?*
- 240 litres of salty water then added into pipe over 5 minutes $\Rightarrow 0.05 m^3/\text{min}$ added to the flow (roughly 10% increase (?))
  May lead to flooding
- Conductivity measured in the bins where output is expected and 2 each side.
- Figure shows the trace from those bins.

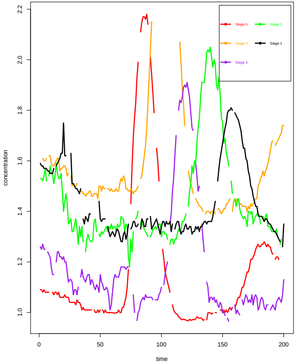


Data treatment

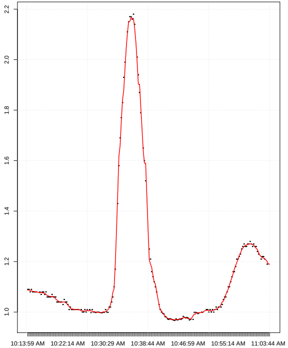
Data were collected using conductivity probes in the bins (stages).

These data were very noisy.

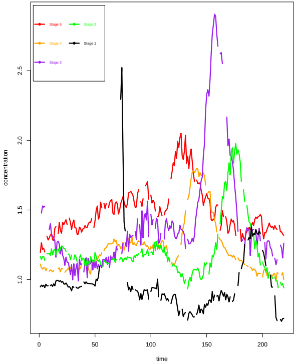
Smoothing (using 6 point moving average) of the data enabled a better picture for each case.



Front stage data

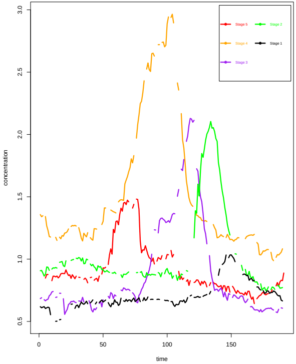


Stage 5

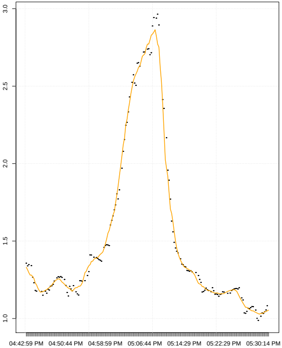


Middle stage data

Stage 5

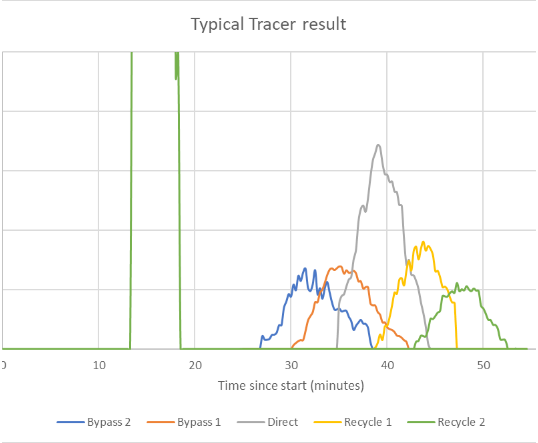


Backend data



Stage 4

Finally looks like ....



"Turbulent" Diffusion of Salt

The spread is very large along the length of the diffuser. Is this salt diffusion?

Consider a "blob" of salty water enters at the spray jet. As it travels downward it interacts with other (non-salty water) perhaps leading to salt diffusion (due to high gradient - see above)

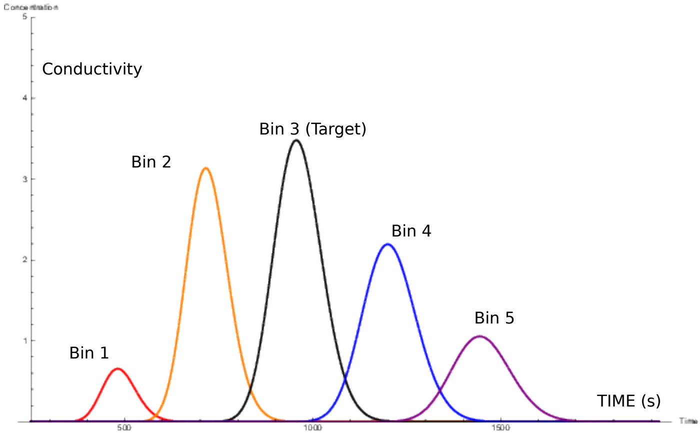
Salt diffusion satisfies, in cylindrical polar coordinates ...

$$\frac{\partial S}{\partial t} = \frac{\alpha_S}{r}\frac{\partial^2 S}{\partial r^2} \text{ which for a point source gives } S(r,t) = \frac{1}{2\alpha_S t} e^{-r^2/4\alpha_S t}.$$

where $\alpha_S = 1.5 \times 10^{-4}\ m^2/s$ (NOT molecular .... $\approx 10^{-9}$)

As the blob moves, $r = \sqrt{(x-ct)^2 + (y-vt)^2}$, and tracking when this arrives at the appropriate bins ....

"Turbulent" Diffusion of Salt



Bins 1 to 5 left to right. Bin 3 is the target ....
Given high salt gradient, this may be sufficent ...

Model 1 - "Ideal flow match"

- The water from each spray jet enters the corresponding bin for that jet.
- The water (juice) is collected and fed into next spray upstream.
- Let $C_k(t), k = 1, 2, 3, ...$ be the concentration in each tray, and $C_0(t)$ be the upstream input concentration, ... then

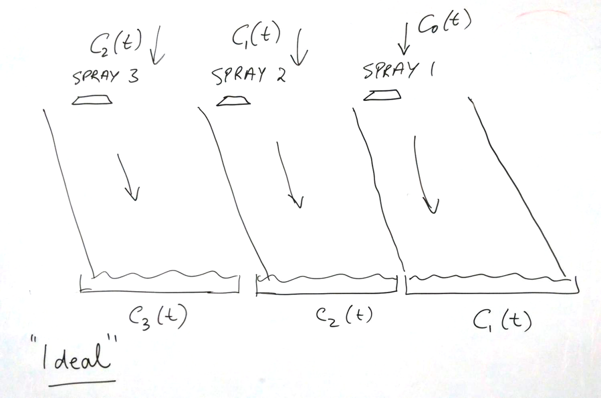
$$\frac{dC_1}{dt} = \beta \left(C_0(t - \tfrac{\kappa}{D}) - C_1(t) \right) \tag{1}$$

$$\frac{dC_2}{dt} = \beta \left(C_1(t - \tfrac{\kappa}{D}) - C_2(t) \right) \tag{2}$$

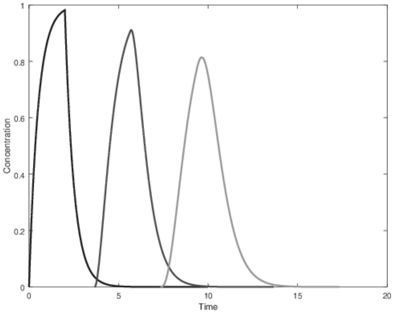
$$\frac{dC_3}{dt} = \beta \left(C_2(t - \tfrac{\kappa}{D}) - C_3(t) \right) \tag{3}$$

$\beta = \frac{q}{V}$, $\kappa =$ permeability, $q=$ mass flux, $V=$tray volume, $D=$fibre depth.

Ideal flow curves



Ideal flow curves



Model 2 - "Recycle"

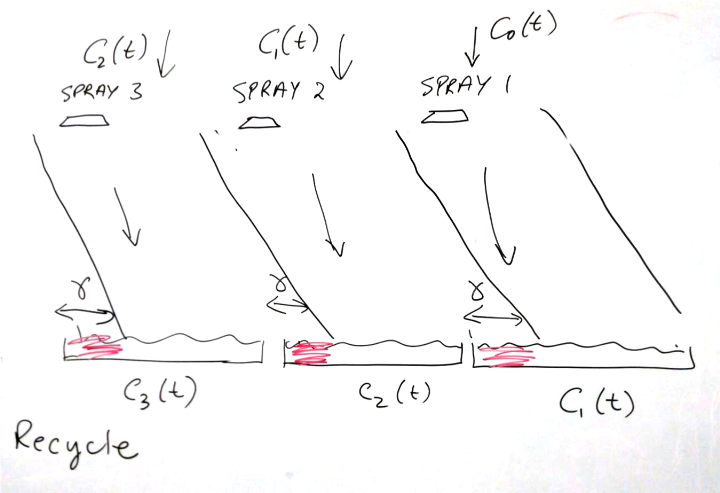
- The water from each spray jet enters the corresponding bin for that jet, but some gets into the same bin as it started in - called Recycling.
- The water (juice) is collected and fed into next spray upstream.
- The inputs to each tray are now modified as ...

$$\frac{dC_1}{dt} = \beta\left((1-\gamma)C_0(t-\tfrac{\kappa}{D}) + \gamma C_1(t-\tfrac{\kappa}{D}) - C_1(t)\right) \quad (4)$$

$$\frac{dC_2}{dt} = \beta\left((1-\gamma)C_1(t-\tfrac{\kappa}{D}) + \gamma C_2(t-\tfrac{\kappa}{D}) - C_2(t)\right) \quad (5)$$

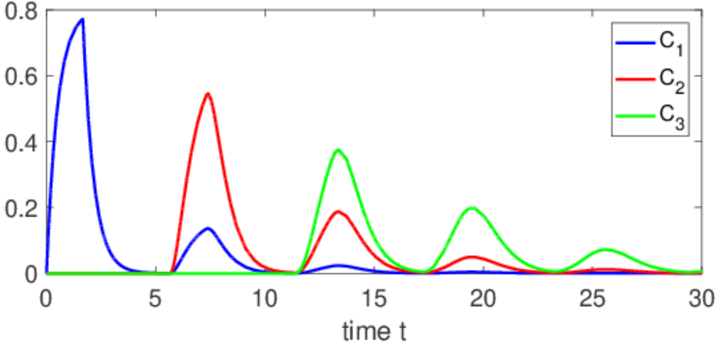
$$\frac{dC_3}{dt} = \beta\left((1-\gamma)C_2(t-\tfrac{\kappa}{D}) + \gamma C_3(t-\tfrac{\kappa}{D}) - C_3(t)\right) \quad (6)$$

Recycle



$C_2(t)$
$C_1(t)$
$C_0(t)$
SPRAY 3
SPRAY 2
SPRAY 1
γ
γ
γ
$C_3(t)$
$C_2(t)$
$C_1(t)$
Recycle

Recycled flow curves - $\gamma = 0.2$



Model 3 - "Bypass"

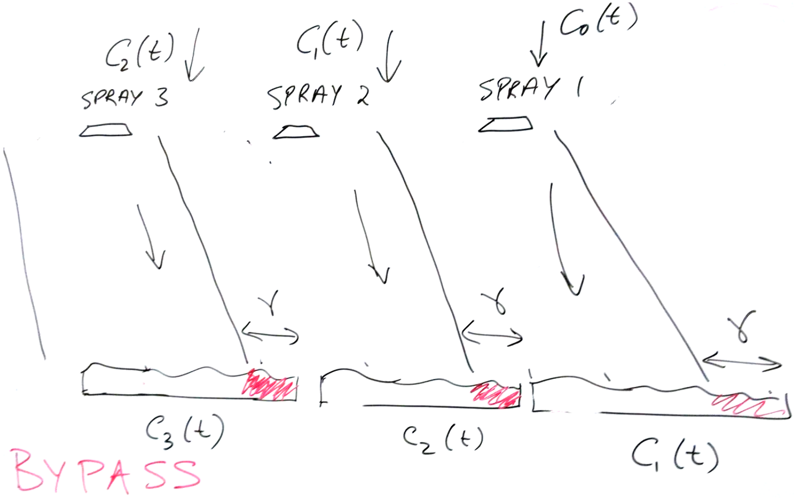
- The water from each spray jet enters the corresponding bin for that jet, but some gets into the more upstream bin and so bypasses a spray.
- The water (juice) is collected and fed into next spray upstream.
- The inputs to each tray are now modified as ...

$$\frac{dC_1}{dt} = \beta\left((1-\gamma)C_0(t-\tfrac{\kappa}{D}) - C_1(t)\right) \quad (7)$$

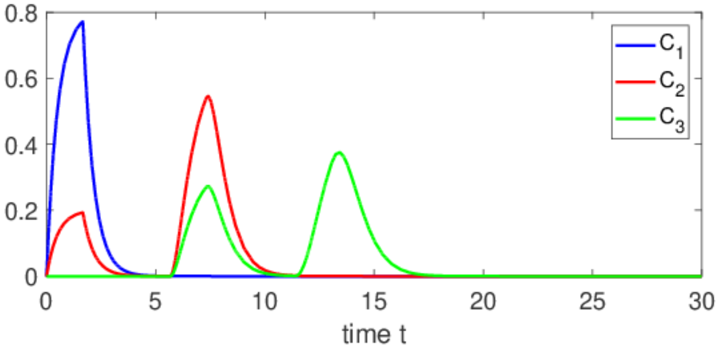
$$\frac{dC_2}{dt} = \beta\left((1-\gamma)C_1(t-\tfrac{\kappa}{D}) + \gamma C_0(t-\tfrac{\kappa}{D}) - C_2(t)\right) \quad (8)$$

$$\frac{dC_3}{dt} = \beta\left((1-\gamma)C_2(t-\tfrac{\kappa}{D}) + \gamma C_1(t-\tfrac{\kappa}{D}) - C_3(t)\right) \quad (9)$$

Bypass curves



Bypass flow curves - $\gamma = 0.2$



What should tracers look like? (Non-diffusive)

- If the flow is perfectly matched, a succession of single bumps separated by the time of travel.
- If there is re-cycling, a second, smaller bump in Bin 1 will appear simultaneous to the primary bump in Bin 2.
- If there is by-pass, a first primary bump will appear at Bin 2 simultaneous to the primary from Bin 1.
- It would seem unlikely that both bypass and recycling will occur in the same run, except in cases very close to ideal, where there is a bit of spread ...

Comments

- Results of experiments tracing salt concentration were examined in an attempt to interpret the results.
- The spread of salt across 5 bins is too great to be caused by the flow
- Suggests the salty water is probably not acting as a tracer.
- Therefore, possible cause of the traces is either flooding (due to the extra liquid input), or...
- Salt diffusion due to turbulent interactions and the high salt content of the fluid *c.f.* surrounds (gives the best comparison).